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Short Note

A solution for TM-mode plane waves incident on a two-dimensional inhomogeneity

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INTRODUCTION

Quantitative interpretation of magnetotelluric (MT) surveys depends at present on the availability of an efficient forward modeling algorithm. To date, two major numerical techniques have been used to obtain the scattered fields from buried inhomogeneities in plane-wave fields: methods solving the governing differential equation which generally uses a finite-element or finite-difference approach, and methods which solve an integral-equation formulation of the problem.

For two-dimensional (2-D) inhomogeneities a solution for incident fields with the electric field parallel to the strike of the inhomogeneity (TE mode solution) was developed by Hohmann (1971) using the integral-equation approach. For a perfect conductor an integral formulation, for surface scattering currents, for the TM mode (magnetic field parallel to the strike of the inhomogeneity) was developed by Parry (1969). General 2-D solutions in the presence of an arbitrary mode plane wave (mixed TE-TM) were obtained by Ryu (1970), Swift (1971), and Rijo (1977) using either a finite-element or finite-difference technique.

To our knowledge, the TM integral-equation solution for the general case has not been presented. The solution presented here thus completes the analysis for the scattering of arbitrary mode plane waves from 2-D inhomogeneities using the integral equation approach. Apart from significant computational advantages in forward modeling of simple geologic bodies for MT analysis, this solution is important for evaluating the results of alternate numerical methods used for more complicated geologic models. It is becoming evident that, for many of the current numerical modeling schemes, there are no convincing checks on the accuracy of the solution. It is imperative, therefore, that several solutions be obtained by different methods and that they be compared until confidence is attained in these solutions.

FORMULATION OF 2-D INTEGRAL EQUATION

Maxwell's equations

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}, \tag{1}$$

and

$$\nabla \times \mathbf{H} = (\sigma + j\omega\varepsilon)\mathbf{E} + \mathbf{J}^s, \tag{2}$$

in the presence of an impressed current source $\mathbf{J}^s$, yield

$$\nabla \times \nabla \times \mathbf{E} - k^2\mathbf{E} = -j\omega\mu\mathbf{J}^s, \tag{3}$$

where

$$k^2 = \omega^2\mu\varepsilon - j\sigma\omega\mu.$$

Introducing a 2-D Green's tensor appropriate for the geometry shown by Figure 1 that satisfies

$$\nabla \times \nabla \times \underset{\sim}{\mathbf{G}}^E(\rho/\rho') - k_n^2(\rho)\underset{\sim}{\mathbf{G}}^E(\rho/\rho') = \underline{\mathbf{I}}\delta(\rho - \rho'), \tag{4}$$

and linearly combining equations (3) and (4) using the procedure described by Weidelt (1975), we obtain the following 2-D integral equation for the electric field $\mathbf{E}(E_x, E_z)$:

$$\mathbf{E}(\rho) = \mathbf{E}^i(\rho) - j\omega\mu \int_S \underset{\sim}{\mathbf{G}}^E(\rho/\rho') \cdot \Delta\sigma(\rho')E(\rho')\,ds. \tag{5}$$

$\mathbf{E}^i(\rho)$ is the incident field at $\rho = (x^2 + z^2)^{1/2}$ that would exist in the absence of the inhomogeneity. The symbol $\underline{\mathbf{I}}$ in equation (4) is a 2-D identity matrix, and $\delta(\cdot)$ is the Dirac delta function. The constant $-j\omega\mu\Delta\sigma$ is defined by $k^2 - k_n^2$, where k_n is the propagation constant representing the layered medium. The term $\Delta\sigma\mathbf{E}$ is called the scattering current (Harrington, 1961).

The numerical solution for $\mathbf{E}$ in equation (5) is initiated by dividing the inhomogeneous region S into a finite number of rectangular cells (Figure 1), such that a constant electric-field intensity can be assumed in each cell. With this assumption the integral equation is reduced to a numerically equivalent matrix form as

$$\underset{\sim}{\mathbf{K}}\mathbf{E} = \mathbf{E}^i \tag{6}$$

where the elements of $\underset{\sim}{\mathbf{K}}$ are given by

$$K_{\ell m} = j\omega\mu\Delta\sigma_m\Gamma_{\ell m}^E + \delta_{\ell m}; \qquad \ell, m = 1, N.$$

The term $\Gamma_{\ell m}^E$ is defined as

$$\Gamma_{\ell m}^E = \int_{S_m} G^E(\rho_\ell/\rho')\,ds,$$

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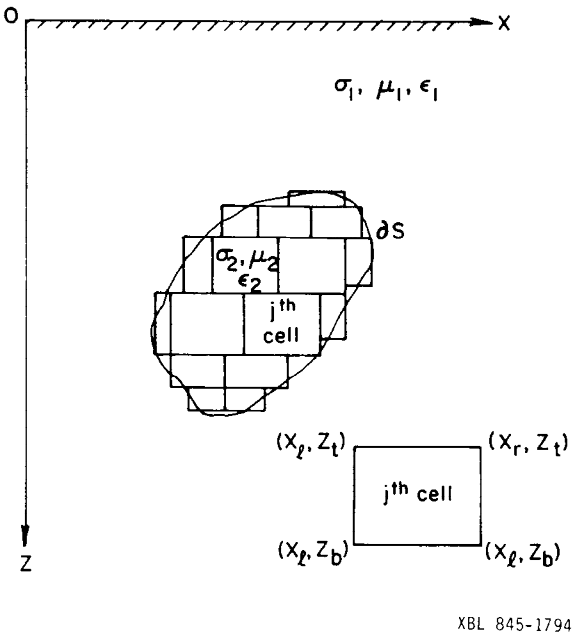

FIG. 1. A simulation of a two-dimensional inhomogeneity by N rectangular cells.

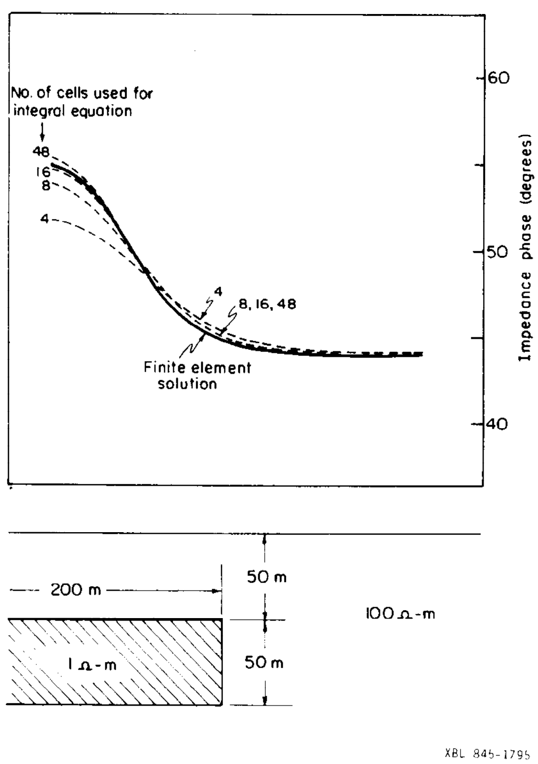

FIG. 2. Convergence test for the integral-equation numerical solution in terms of impedance phase. The dotted lines represent integral-equation solutions with a varying number of cells used.

and it symbolically means that it is the integral of G^E over the area occupied by the mth cell evaluated at the center of the ℓth cell. The symbol $\delta_{\ell m}$ is the 2-D Kronecker delta function, and N is twice the total number of cells used.

Once we find the electric fields within the inhomogeneity by solving the system of equations [equation (6)], the electric fields at positions away from the inhomogeneity may be calculated using essentially the same equation [equation (5)]. In this case the field position ρ is usually on or above the earth-air interface, and therefore different Green's functions should be used. For the magnetic fields, we modify equation (5) into, by virtue of equation (1),

$$\mathbf{H}(\rho) = \mathbf{H}^i(\rho) + \int_S \nabla \times \underset{\sim}{\mathbf{G}}^E(\rho/\rho') \cdot \Delta\sigma(\rho')\mathbf{E}(\rho')\, ds. \tag{7}$$

The computations of Green's functions $\underset{\sim}{\mathbf{G}}^E$ and $\nabla \times \underset{\sim}{\mathbf{G}}^E$ frequently require time-consuming numerical integrations in harmonic space ($k_x \rightarrow x$). A detailed description of those necessary Green's functions and numerical integrations involved was given by Lee and Morrison (1984). The numerical integration usually takes more than half of the total computing time required for a final solution.

NUMERICAL RESULTS AND CONCLUSION

The numerical solution developed here is compared to the one obtained by Ryu (1970) using the finite-element method. The model used is a half-space in which a rectangular conductor of 200 m × 50 m is buried 50 m deep to the top of the conductor. The resistivities used are 1 Ω · m for the conductor and 100 Ω · m for the half-space. At the frequency of 8 Hz, the finite-element solution was obtained on a grid in which the conductor is simulated by 16 cells of equal size (25 m × 25 m). At this frequency the cell size used for the finite-element solution is considered fine enough, with its skin depth of 177 m, for the accurate numerical solution. Compared to the finite-element solution, a number of integral-equation solutions were obtained using increasingly more cells for the conductor. The result was plotted (Figure 2) in terms of the impedance phase. The integral-equation solution converges to the finite-element solution as the number of cells used increases from 4 cells to 48 cells. The convergence rate is relatively slow compared to the one for the TE-mode solution (Hohmann, 1971), in which a cell size of a quarter of the skin depth would result in a reasonably good numerical solution. The slow convergence of the TM-mode numerical solution seems to have resulted from the assumption we used, in which electric fields are assumed constant in a cell. Unlike the TE-mode situation where there is no charge, the discontinuous normal electric fields at cell boundaries create charges. The Hertz potential of a constant current, by which the electromagnetic field is computed here, does not include potentials due to thus created charges and eventually affects the quality of the numerical solution. The effect of the cell boundary charges can be somewhat reduced through the approach in which an explicit scalar potential is employed to represent fields due to charges. Hohmann and Ting (1978) used a scheme by which the concentrated boundary charges are uniformly distributed over a volume extending from the center of one cell to the center of the next cell.

The next model compared is a conductive dike with varying dip angles as shown in Figure 3. A dipping boundary of the conductor is simulated by stacking small rectangular conduc-

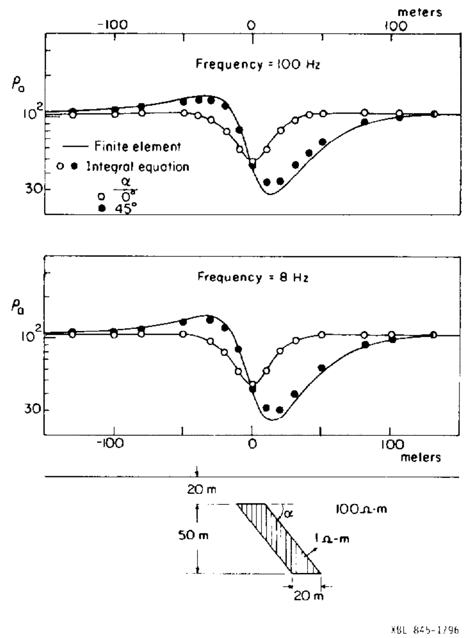

FIG. 3. Comparison between solutions for a dipping conductor. The solid line represents apparent resistivity obtained using finite-element method. α is the dip of the conductor simulated.

tors in a way that preserves the overall dip angle. For frequencies of 8 and 100 Hz, the finite-element solution and the integral-equation solution result in identical apparent resistivity profiles when the conductor (20 m × 50 m) is vertical. With the dip angle of 45 degrees, however, a maximum of 10 percent difference in peak-to-peak apparent resistivity is observed for both frequencies. For the integral-equation solutions the conductor was simulated by 10 cells of equal size—10 m × 10 m. Considering the slow convergence rate illustrated by Figure 2, the numerical integral-equation solution for the dipping conductor, with its increased boundary surface, may not have reached its full convergence.

Using the integral equation for the modeling of electromagnetic scattering limits work to a model finite in extent in an otherwise layered half-space. It is unfortunate that the available Green's functions are limited to those for the layered half-space. Ironically, however, this geometrical restriction offers the integral-equation technique its major advantage; it is necessary to solve for the scattering current only in the inhomogeneous region.

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